Geometry
For
Middle School Teachers

A Professional Development Program to Implement the 2001 Virginia Standards of Learning

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Office of Middle Instructional Services
Virginia Department of Education
P.O. Box 2120
Richmond, Virginia 23218-2120

Superintendent of Public Instruction
Jo Lynne DeMary

Assistant Superintendent for Instruction
Patricia I. Wright

Office of Middle Instructional Services
James Firebaugh, Director
Lois Williams, Middle Mathematics Specialist

Office of Elementary Instructional Services
Karen Grass, Elementary Mathematics Specialist

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Margie Mason, Associate Professor
The College of William and Mary
Williamsburg, Virginia

Bruce Mason, Consultant
Williamsburg, Virginia

Dana Johnson, Instructor
The College of William and Mary
Williamsburg, Virginia

Carol L. Rezba, Assistant Professor
Longwood University

Christa Southall
Stafford County

Introduction

The updated *Geometry for Middle School Teachers* is a professional development program designed to assist teachers in implementing the 2001 Virginia Mathematics Standards of Learning. This professional development program provides a sample of meaningful and engaging activities correlated to the geometry strand of the grades 6-8 Mathematics Standards of Learning.

The purpose of the professional development program is to enhance teachers' content knowledge and their use of instructional strategies for teaching the geometry strand of the Mathematics Standards of Learning. Teachers will learn about the van Hiele model for the development of geometric thought and how this can be used to guide instruction and classroom assessment. Through explorations, problem solving, and hands-on experiences, teachers will engage in discussions and activities that address many of the dimensions of geometry including spatial relationships, properties of geometric figures and solids, constructions, geometric modeling, geometric transformations, coordinate geometry, the geometry of measurement, informal geometric reasoning, and geometric connections to the physical world. Teachers will explore two- and three-dimensional figures, paper folding, tessellations, and the use of other manipulatives and technology to develop geometric understanding. Through these activities, it is anticipated that teachers will develop new techniques that assist in increasing student achievement in their classrooms.

Designed to be presented by teacher trainers, this professional development program includes directions for the trainer, as well as the activity sheets for participants. An addendum to the module includes video segments of the van Hiele levels. These video segments portray students engaged in assessment tasks and can be used to discuss students' level of development of geometric thought. Directions for the assessment tasks are also included.

Trainers should adapt the materials to best fit the needs of their audience; adding materials that may be more appropriate for their audience and eliminating materials that have been used in previous training sessions. All materials in this document may be duplicated and distributed as desired for use in public schools in Virginia.

The training programs are organized into five modules that may be offered by school divisions for teacher licensure renewal points or for a one-credit graduate course, when university credit can be arranged.

GLOSSARY

Acute Angle	An angle with a measure greater than 0 degrees but less than 90 degrees.
Acute Triangle	A triangle with three acute angles (or no angle measuring 90 degrees or greater).
Adjacent Sides	Two sides of a polygon with a common vertex.
Angle	Two rays that share an endpoint.
Arc	Part of a circle.
Area	The amount of surface in a region or enclosed within a boundary. Area is measured in square units such as square feet or square centimeters.
Attribute	A characteristic possessed by an object. Characteristics include shape, color, size, length, weight, capacity, area, etc.
Base of a Solid	A plane figure. If the solid is a cylinder or prism, there are two bases that are parallel and congruent.
Centimeter	A metric unit of length equal to one-hundredth of one meter.
Circle	A closed curve with all points in one plane and equidistant from a fixed point (the center).
Circumference	The length of the boundary of a circular region. The circumference can be computed by multiplying the diameter by *pi* (π), a number a little more than 3.14.
Concentric Circles	Two or more circles that have the same center and different radii.
Cone	A three-dimensional figure with one curved surface, one flat surface (usually circular), one curved edge, and one vertex.
Congruent	Having exactly the same size and shape. Congruent polygons have their corresponding angles congruent and corresponding sides congruent.
Coordinate System	A reference system for locating and graphing points. In two dimensions, a coordinate system usually consists of a horizontal axis and a vertical axis, which intersect at the origin. Each point in the plane is located by its horizontal distance and vertical distance from the origin. These distances, or coordinates, form an ordered pair of numbers.
Cube	A solid figure in which every face is a square and every edge is the same length.

Cubic Foot	The volume of a cube that is one foot wide, one foot high, and one foot long.
Cubic Unit	A unit of measure that has a length of one unit, a width of one unit, and a height of one unit used to measure volume. Examples are cubic inches, cubic centimeters, etc.
Cylinder	A solid figure formed by two congruent parallel circles joined by a curved surface.
Decagon	A polygon with ten sides. A regular decagon has ten congruent sides and ten congruent angles.
Diagonal	A line segment that joins two non-adjacent vertices of a polygon or polyhedron.
Diameter	A line segment passing through the center of a circle or sphere and connecting two points on the circle or sphere.
Diamond	See *Rhombus.*
Dimension	The number of coordinates used to express a position.
Dodecagon	A polygon with twelve sides. A regular dodecagon has twelve congruent sides and twelve congruent angles.
Dodecahedron	A polyhedron with twelve faces. All faces of a regular dodecahedron are congruent, regular pentagons.
Edge	A line segment where two faces of a three-dimensional figure intersect.
Endpoint	The point(s) at the end of a ray or line segment.
Equilateral Triangle	A triangle with three congruent sides. Each angle measures 60 degrees.
Face	A plane figure that serves as one side of a solid figure.
Flip	See *Reflection.*
Geometry	The branch of mathematics that deals with the position, size, and shape of figures.
Grid	A network of horizontal and vertical lines that intersect to form squares or rectangles.
Hemisphere	Half of a sphere, formed by making a plane cut through the center of a sphere.

Heptagon	A polygon with seven sides. A regular heptagon has seven congruent sides and seven congruent angles.
Hexagon	A polygon with six sides. A regular hexagon has six congruent sides and six congruent angles.
Hexahedron	A polyhedron with six faces. All faces of a regular hexahedron are congruent squares. A regular hexahedron is a cube.
Hypotenuse	The side opposite the right angle of a right triangle. The hypotenuse is the longest side of a right triangle.
Icosahedron	A polyhedron with twenty faces. All faces of a regular icosahedron are congruent, equilateral triangles.
Isosceles Triangle	A triangle with at least two congruent sides and two congruent angles. An equilateral triangle is a special case of an isosceles triangle.
Kite	A convex quadrilateral with two distinct pairs of adjacent, congruent sides.
Line	A set of points that forms a straight path extending infinitely in two directions. Lines are often called “straight lines” to distinguish them from curves, which are often called “curved lines.” Part of a line with two endpoints is called a “line segment.”
Line of Symmetry	A line dividing a two-dimensional figure into two parts that are mirror images of each other.
Line Segment	A part of a line. A line segment has two endpoints and a finite length.
Network	A diagram consisting of arcs (branches) connecting points or nodes (junctions). A network may represent a real-world situation, such as a road system or an electronic circuit. Sometimes the nodes are called vertices.
Node	A point in a network at the end of an arc or at the junction of two or more arcs.
Nonagon	A polygon with nine sides. A regular nonagon has nine congruent sides and nine congruent angles.
Obtuse Angle	An angle that is greater than 90 degrees but less than 180 degrees; that is, between a right angle and a straight line.
Obtuse Triangle	A triangle that has one obtuse angle.
Octagon	A polygon with eight sides. A regular octagon has eight congruent sides and eight congruent angles.

Octahedron	A polyhedron with eight faces. All faces of a regular octahedron are congruent, equilateral triangles.
Opposite Angles	In a quadrilateral, angles that do not have a common side; non-adjacent angles.
Parallel Lines	Lines lying in the same plane that are always the same distance apart.
Parallelogram	A quadrilateral with both pairs of opposite sides parallel. Opposite angles are congruent.
Pentagon	A polygon with five sides. A regular pentagon has five congruent sides and five congruent angles.
Perimeter	The distance around a figure.
Perpendicular	At right angles.
pi (π)	The ratio of the circumference of a circle to its diameter. This ratio is the same for every circle. Its value, which is found by dividing the circumference by the diameter, is a little more than 3.14.
Pie graph	A circle marked into sectors. Each sector shows the fraction represented by one category of data. Pie graphs are also called circle graphs.
Plane	A flat surface extending infinitely in all directions.
Plane Figure	In geometry, a closed two-dimensional figure that lies entirely in one plane. (Polygons and circles are examples of plane figures. An arc is not a plane figure because it is not closed.)
Point	The smallest geometric unit. A position in space, often represented by a dot.
Polygon	A simple, closed, plane figure formed by straight sides.
Polyhedron	A three-dimensional figure in which all the surfaces are polygons.
Prism	A polyhedron with at least one pair of opposite faces that are parallel and congruent. Corresponding edges of these faces are joined by rectangles or parallelograms.
Pyramid	A polyhedron with any polygon for its base. The other faces are triangles that meet at a point or vertex.
Quadrilateral	A polygon with four sides.

Ray	A set of points that form a straight path extending infinitely in one direction. A ray has one endpoint.
Rectangle	A parallelogram with four right angles. Opposite sides are congruent and parallel.
(Right) Rectangular Prism	A solid figure in which all six faces are rectangles. Opposite faces are parallel and congruent.
Reflection	A transformation of a geometric figure that results in a mirror image of the original.
Regular Polygon	A polygon that has congruent sides and congruent angles.
Regular Polyhedron	A polyhedron with congruent faces that are regular polygons.
Rhombus	A parallelogram with four congruent sides. Opposite angles are congruent.
Right Angle	An angle that is one-fourth of a full turn. A right angle measures 90 degrees.
Right Triangle	A triangle that has one right angle.
Scalene Triangle	A triangle with no sides congruent.
Semicircle	An arc that is one-half of a circle.
Similar	Figures that have the same shape but not necessarily the same size. Similar polygons have corresponding angles congruent and corresponding sides in proportion. Congruent is a special case of similar where the ratio of the corresponding sides is 1-1.
Slide	See *Translation*
Solid Figure	A closed, three-dimensional figure.
Sphere	A three-dimensional figure formed by a set of points that are all the same distance from a fixed point called the center.
Square	A rectangle with congruent sides.
Square Unit	A unit of measure that has a length of one unit and a width of one unit used to measure area. Examples are square inches, square centimeters, acres, etc.
Surface	Part or all of the boundary of a solid. A surface may be flat or curved. (For example, a cone has one flat surface and one curved surface.)

Symmetry	a. If a figure can be folded along a line so that the two halves match exactly, then the figure has line symmetry. b. If a figure can be turned less than 360 degrees about a point and fit exactly on itself, then a figure has rotational symmetry.
Tessellation	An arrangement of plane figures (usually congruent figures) to cover a surface without overlapping or leaving any gaps.
Tetrahedron	A polyhedron with four triangular faces. A tetrahedron is a triangular pyramid.
Three-Dimensional	Relating to objects that have length, width, and depth. Solid figures such as polyhedra, cones, and spheres are three-dimensional.
Transformation	Changing a geometric figure, according to a rule. Examples of transformations are reflection, rotation, and translation.
Translation	A transformation in which a geometric figure is formed by moving every point on a figure the same distance in the same direction.
Trapezoid	A quadrilateral with exactly one pair of parallel sides.
Triangle	A polygon with three sides.
Triangular Prism	A prism in which the bases are triangles.
Two-Dimensional	Relating to figures that have length and width but not depth. Figures such as polygons and circles are two-dimensional.
Vertex	a. A point at which two line segments, lines, or rays meet to form an angle. b. A point on a polyhedron where three or more faces intersect. c. The point opposite the base of a cone or pyramid. The plural of vertex is vertices.

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Session V

Geometry Standards of Learning Kindergarten through Eighth Grade

* indicates a related Standard of Learning from the Measurement Strand

K.11 The student will identify, describe, and draw two-dimensional (plane) geometric figures (circle, triangle, square, and rectangle).

K.12 The student will describe the location of one object relative to another (above, below, next to) and identify representations of plane geometric figures (circle, triangle, square, and rectangle) regardless of their position and orientation in space.

K.13 The student will compare the size (larger, smaller) and shape of plane geometric figures (circle, triangle, square, and rectangle).

1.15 The student will describe the proximity of objects in space (*near, far, close by, below, above, up, down, beside,* and *next to).*

1.16 The student will draw, describe, and sort plane geometric figures (triangle, square, rectangle, and circle) according to number of sides, corners, and square corners.

1.17 The student will identify and describe objects in his/her environment that depict plane geometric figures (triangle, rectangle, square, and circle).

2.20 The student will identify, describe, and sort three-dimensional (solid) concrete figures, including a cube, rectangular solid (prism), square pyramid, sphere, cylinder, and cone, according to the number and shape of the solid's faces, edges, and corners.

2.21 The student will identify and create figures, symmetric along a line, using various concrete materials.

2.22 The student will compare and contrast plane and solid geometric shapes (circle/sphere, square/cube, and rectangle/rectangular solid).

3.18 The student will analyze two-dimensional (plane) and three-dimensional (solid) geometric figures (circle, square, rectangle, triangle, cube, rectangular solid [prism], square pyramid, sphere, cone, and cylinder) and identify relevant properties, including the number of corners, square corners, edges, and the number and shape of faces, using concrete models.

3.19 The student will identify and draw representations of line segments and angles, using a ruler or straightedge.

3.20 The student, given appropriate drawings or models, will identify and describe congruent and symmetrical, two-dimensional (plane) figures, using tracing procedures.

4.13* The student will
a) identify and describe situations representing the use of perimeter and area; and
b) use measuring devices to find perimeter in both standard and nonstandard units of measure. (*Measurement strand)

4.14 The student will investigate and describe the relationships between and among points, lines, line segments, and rays.

4.15 The student will
a) identify and draw representations of points, lines, line segments, rays, and angles, using a straightedge or ruler; and
b) describe the path of shortest distance between two points on a flat surface.

4.16 The student will identify and draw representations of lines that illustrate intersection, parallelism, and perpendicularity.

4.17 The student will
a) analyze and compare the properties of two-dimensional (plane) geometric figures (circle, square, rectangle, triangle, parallelogram, and rhombus) and three-dimensional (solid) geometric figures (sphere, cube, and rectangular solid [prism]);
b) identify congruent and noncongruent shapes; and
c) investigate congruence of plane figures after geometric transformations such as reflection (flip), translation (slide) and rotation (turn), using mirrors, paper folding, and tracing.

4.18 The student will identify the ordered pair for a point and locate the point for an ordered pair in the first quadrant of a coordinate plane.

5.8* The student will describe and determine the perimeter of a polygon and the area of a square, rectangle, and right triangle, given the appropriate measures. (*Measurement strand)

5.10* The student will differentiate between perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation. (*Measurement strand)

5.13* The student will measure and draw right, acute, and obtuse angles and triangles, using appropriate tools. (*Measurement strand)

5.14 The student will classify angles and triangles as right, acute, or obtuse.

5.15 The student, using two-dimensional (plane) figures (square, rectangle, triangle, parallelogram, rhombus, kite, and trapezoid) will
a) recognize, identify, describe, and analyze their properties in order to develop definitions of these figures;
b) identify and explore congruent, noncongruent, and similar figures;
c) investigate and describe the results of combining and subdividing shapes;
d) identify and describe a line of symmetry; and
e) recognize the images of figures resulting from geometric transformations such as translation (slide), reflection (flip), or rotation (turn).

5.16 The student will identify, compare, and analyze properties of three-dimensional (solid) geometric shapes (cylinder, cone, cube, square pyramid, and rectangular prism).

Grade 6

6.11* The student will determine if a problem situation involving polygons of four or fewer sides represents the application of perimeter or area and apply the appropriate formula. (*Measurement strand)

6.12* The student will
a) solve problems involving the circumference and/or area of a circle when given the diameter or radius: and
b) derive approximations for pi (B) from measurements for circumference and diameter, using concrete materials or computer models. (*Measurement strand)

6.13* The student will
a) estimate angle measures, using 45°, 90°, and 180°, referents, and use the appropriate tools to measure the given angles; and
b) measure and draw right, acute, and obtuse angles and triangles. (*Measurement strand)

6.14 The student will identify, classify, and describe the characteristics of plane figures, describing their similarities, differences, and defining properties.

6.15 The student will determine congruence of segments, angles, and polygons by direct comparison, given their attributes**.** Examples of noncongruent and congruent figures will be included.

6.16 The student will construct the perpendicular bisector of a line segment and an angle bisector.

6.17 The student will sketch, construct models of, and classify solid figures (rectangular prism, cone, cylinder, and pyramid).

Grade 7

7.7 The student, given appropriate dimensions, will
a) estimate and find the area of polygons by subdividing them into rectangles and right triangles; and
b) apply perimeter and area formulas in practical situations. (*Measurement strand)

7.8 The student will investigate and solve problems involving the volume and surface area of rectangular prisms and cylinders, using concrete materials and practical situations to develop formulas. (*Measurement strand)

7.9 The student will compare and contrast the following quadrilaterals: parallelogram, rectangle, square, rhombus, and trapezoid. Deductive reasoning and inference will be used to classify quadrilaterals.

7.10 The student will identify and draw the following polygons: pentagon, hexagon, heptagon, octagon, nonagon, and decagon.

7.11 The student will determine if geometric figures — quadrilaterals and triangles — are similar and write proportions to express the relationships between corresponding parts of similar figures.

7.12 The student will identify and graph ordered pairs in the four quadrants of a coordinate plane.

7.13 The student, given a polygon in the coordinate plane, will represent transformations — rotation and translation — by graphing the coordinates of the vertices of the transformed polygon and sketching the resulting figure.

Grade 8

8.6* The student will verify by measuring and describe the relationships among vertical angles, supplementary angles, and complementary angles and will measure and draw angles of less than 360°.(*Measurement strand)

8.7* The student will investigate and solve practical problems involving volume and surface area of rectangular solids (prisms), cylinders, cones, and pyramids. (*Measurement strand)

8.8 The student will apply transformations (rotate or turn, reflect or flip, translate or slide, and dilate or scale) to geometric figures represented on graph paper. The student will identify applications of transformations, such as tiling, fabric design, art, and scaling.

8.9 The student will construct a three-dimensional model, given the top, side, and/or bottom views.

8.10 The student will

a) verify the Pythagorean Theorem, using diagrams, concrete materials, and measurement; and
b) apply the Pythagorean Theorem to find the missing length of a side of a right triangle when given the lengths of the other two sides.